About submission of the operator such as Cauchy in classes of traces of Sobolevsky of functions on Jordan circuits

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Abstract **-To be resulted structural construction of the operator such as Cauchy (analog of an integral of Cauchy) on Jordan circuits not being locally rectifiable. In classes of traces of Sobolevsky of functions to be plotted realization of the operator such as Cauchy and his limitation is proved.**

I. INTRODUCTION

Under the integral or the operator such as Cauchy imply an integral of a kind

$$\frac{1}{2\pi i}\int_{\Gamma}\frac{f(t)\,dt}{t-z}\equiv\frac{1}{2}S(f/z) \qquad (1)$$

perceived in sense of a principal value. This operator determines an analytic function in a complex plane (c.p.). $z\notin\Gamma$ Also it is used for submissions of problem solving of mathematical physics. Submission (1) has properties of the operator, therefore in 50 years the problem of construction of the operator such as Cauchy on locally nonrectifiable circuits has been formulated. The partial solution of this problem in the beginning has been given one of writers in activities [1], [2].

The operator such as Cauchy $S(\varphi)$, (1) , is under construction in [1], [2] as follows. Let closed Jordans the circuit Γ (the holomorphic image of a circle) divides c.p. on internal D^+ and external D^- domains and let $\varphi(t), t\in\Gamma$ continuous functions on Γ.

Let exist unique piecewise-analytic in $D^\pm$ function $\Phi^\pm(z),\left(z\in D^\pm\right)$, such that there are continuous limiting values

$$\Phi^\pm(t)=\lim_{z\to t}\Phi^\pm(z), \qquad z\notin\Gamma \qquad (2)$$

also are fulfilled conditions

$$\Phi^+-\Phi^-=\varphi(t), \quad t\in\Gamma \qquad \Phi^-(\infty)=0 \qquad (3)$$

Then the operator such as Cauchy $S(\varphi/t)$ on Γ is determined by submission

$$S(\varphi/t)=\left(\Phi^+(t)+\Phi^-(t)\right) \qquad (4)$$

Thus for $z\in D^\pm$ by definition we suppose

$$\Phi^\pm(z)=\frac{1}{2}S(\varphi/z), z\in D^\pm \qquad (5)$$

(Such $\Phi^\pm(z)$ exist according to (2) and are unique)

Lemma 1 (about structure of the operator S): Let properties (2) - (5) are executed. Then:

a) The analog of formula Sohockogo occurs

$$\Phi^\pm(t)=\lim_{z\to t\in\Gamma}\Phi^\pm(z)=\pm\frac{1}{2}\varphi(t)+\frac{1}{2}S(\varphi/t) \qquad (6)$$

b) Property of involutive is executed

$$S^2(\varphi)=S(S(\varphi))\equiv\varphi \text{ (ò.å. } S\equiv S^{-1}) \qquad (7)$$

The proof:

The formula (6) follows directly from (3) and (4), moreover properties (2) - (4) equivalents to property (6).

For the proof of property (7) we shall consider an event, when $\varphi(t)\equiv\Phi^+(t)$ and $\Phi^-(t)\equiv-\varphi(t)$. In case of the former supposing according to (5) $\Phi^\pm(z)=\frac{1}{2}S\left(\pm\Phi^\pm/z\right), z\in D^\pm$ and taking into account (2) and (5) we receive a frontier property of analytic functions.

$$\Phi^+(t)=\frac{1}{2}\Phi^+(t)+\frac{1}{2}S\left(\Phi^+/t\right)\Leftrightarrow\Phi^+(t)\equiv S\left(\Phi^+(t)\right)$$

$$\Phi^-(t)=\frac{1}{2}\Phi^-(t)-\frac{1}{2}S(\Phi^-/t)\Leftrightarrow\Phi^-(t)\equiv-S(\Phi^-(t)), \quad (8)$$

$$\Phi^-(\infty)=0$$

Fair for any piecewise-analytic in $D^\pm$ function. From here with allowance for (3) and (4) it is discovered

$$\varphi(t)=\Phi^+(t)-\Phi^-(t)\equiv S\left(\left(\Phi^+-\Phi^-\right)/t\right)=S\left(S(\varphi/t)\right)$$

The lemma 1 is proved .

The remark 1. Submission (4) is determined by the unique solution of a problem on gallop (3) and fulfilment of properties of boundary conduct (2). Therefore submission (4) does not depend on rectifiable boundaries Γ of domain D^+. For this reason we can consider, that correctly following definition.

Definition (the operator such as Cauchy). Let a restricted singly connected domain D^+ in c.p. z It is limited a Jordan curve $\Gamma = \partial D^+$ (not necessarily rectifiable!). And let for a continuous function $\varphi(t)$, $t \in \Gamma$ exists unique piecewise-analytic functions $\Phi^{\pm}(z), z \in D^{\pm}$ in the domains $D^{\pm}$ $\left(D^+ \cup \Gamma \cup D^- = C^1\right)$, obeying (2) and (3). Then submissions (4) - (5) determine the operator such as Cauchy $S(\varphi/z), z \in C^1$ operational on function φ.

From this definition and a lemma 1 it is received

Inquest 1. In an event of a locally rectifiable circuit Γ the identity (1) is executed.

The remark 2. If instead of (4) to take

$$S(\varphi/t) = \frac{1}{2}\left(\Phi^+(t) - \Phi^-(t)\right)$$

Then instead of (1) we shall receive

$$S(\varphi/t) = \frac{1}{2\pi i}\int_{\Gamma} \frac{\varphi(t)\,dt}{t-z}$$

And property of involutive (7) will become

$$2S\left(2S(\varphi)\right) \equiv \varphi$$

That submissions (3) and (4) could be applied efficiently to problem solving mathematical physics (for example to write the operator of Cauchy on a fractal curve Kokh) it is required to decide a following problem.

To find a class of circuits and functions, the given on these circuits in which operator (4), S has manifestative submission and is the restricted operator.

In the given activity the solution of this problem ground properties of the operator S content in activities of one of writers [1], [2] is stated.

II. THE MAIN OUTCOMES

Let's mark out $W_p^1(G_R)$ - a class of the generalized in sense Sobolev of the functions having first generalized derivative, summable with the degree $p > 2$ in domain. Let $\overset{0}{W}{}_p^1(G)$ subclass in $W_p^1(G_R)$ with the carrier of century $G \subset G_R$. We shall remind, that according to embedding Sobolev's theorem $\overset{0}{W}{}_p^1(G) \subset C_\alpha$, where C_α - a class Gelder's functions, where $\alpha < \frac{p-2}{p}$.

Let's consider such Jordan a circuit $\Gamma = \partial D^+$, which have property:

$$\overset{0}{W}{}_p^1\left(D^+\right) \subset \overset{0}{W}{}_p^1\left(C^1\right), p > 2$$

Class of such curves Γ we shall mark out Λ.

Let's speak, that $\varphi(t)$, the given on Γ belong to a class of tracks of Sobolevsky of functions if exists

$$\tilde{\varphi} \in W_p^1(G_R) \Rightarrow \tilde{\varphi}\big|_\Gamma = \varphi .$$

In these suppositions, using lemmas 1 and 2 from [2] we come to following outcome.

The theorem 1 (About realization of the operator S). Let a Jordan circuit $\Gamma \in \Lambda$, $\Gamma = \partial D^+$. Then for any $\varphi \in SW_p^1(G_R), G_R \supset D^+$ the operator $S(\varphi)$, (4), (5) is determined, which is represented by the way

$$S(\varphi/t) = \varphi(t) - 2T\left(\tilde{\varphi}_{\bar{\xi}}/t\right), t \in \Gamma \qquad (9)$$

$$S(\varphi/t) = \tilde{\varphi}(z) - 2T\left(\tilde{\varphi}_{\bar{\xi}}/z\right), z \in D^+ \qquad (10)$$

$$S(\varphi/t) = -2T\left(\tilde{\varphi}_{\bar{\xi}}/z\right), z \in D^- \qquad (11)$$

Where $\tilde{\varphi}$ - any function of a class $W_p^1(G_R)$ with property $\tilde{\varphi}\big|_\Gamma = \varphi, \tilde{\varphi}_{\bar{\xi}} = \frac{\partial}{\partial \bar{\xi}}\varphi(\xi)$, and

$$T(f/z) = -\frac{1}{\pi}\iint_{D^+} \frac{f(\xi)\,d\xi \wedge d\bar{\xi}}{\xi - z}, z \in C^1$$

Let's remark, that submissions (9) - (11) are invariant concerning selection of prolongation $\tilde{\varphi}$ in $C^1 \setminus \Gamma$ function $\varphi(t)$ of the given on Γ. It follows that for any $\varphi \in \overset{0}{W}{}_p^1\left(D^+\right)$ submission is fair

$$T\left(\varphi_{\bar{\xi}}/z\right) \equiv \varphi(z), z \in C^1 .$$

Therefore changing $\tilde{\varphi}$ in a right member of equalities (9) - (11) on function $\tilde{\varphi} + \varphi$ will not change values of right members.

The factor the space of traces is identifiable

$$SW_p^1(\Gamma) = \overset{0}{W}{}_p^1(G_R) \setminus \overset{0}{W}{}_p^1\left(D^+\right) \qquad (12)$$

It to become Banach's if to enter the norm

$$\|F\|_S = \inf_{f \in \overset{0}{W}{}_p^1\left(D^+\right)} \left\|\tilde{f} - f\right\|_{W_p^1(G_R)} \qquad (13)$$

where $\tilde{f}$ - the fixed element of the class of contiguity F. By virtue of even convexity $W_p^1(G_R)$ the precise greatest lower bound in (12) is reached on a unique element $f_0 \in \overset{0}{W}{}_p^1\left(D^+\right)$.

The theorem 2 (about limitation S). The operator S introduced (9) is linear restricted in the norm (13) by operator

$$S : SW_p^1(\Gamma) \to SW_p(\Gamma) .$$

The proof is based on assessments of works [2].

III. BREEDINGS

We receive submission of the operator such as Cauchy (9) - (11) fair for Jordan curves $\Gamma = \partial D^{+}$ (not necessarily locally rectifiable) conceding prolongation of a class $\overset{\circ}{W}{}_p^1(C^1)$. This submission solves a problem about gallop (3) and recovers piecewise-analytic function to $D^{\pm}$ according (10) and (11). In an event locally a rectifiable Jordan curve Γ submission (9) - (11) will reduce to an integral such as Cauchy (1) and can be utilized for numerical realization. In this case we have the new numerical method of calculation of an integral such as Cauchy, is expedient distinguished from known methods offered recen

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