Iterative reconstruction algorithms in optical tomography in frequency domain case

A.N. Bondarenko[1], D.N. Ivanov

NSTU
630092, Novosibirsk, K.Marks, 20, Russia, E-mail: unpomag@ngs.ru

Abstract - **We present a review of methods for the forward and inverse problems in optical tomography. We limit ourselves to the highly scattering case found in applications in medical imaging, and to the problem of absorption and scattering reconstruction. We concentrate on the use numerical method based on finite elements.**

I. INTRODUCTION

Optical tomography has come to mean the use of low-energy visible or near infra-red light to probe highly scattering media, in order to derive qualitative or quantitative images of the optical properties of these media. Of the potential applications, the one that has received a great deal of attention is medical imaging, where optical tomography is hoped to be a low-cost alternative or complement to existing medical imaging technology, with the particular advantage of providing *functional* as opposed to anatomical information. The name is something of a misnomer since the term already exists to describe the use of light to image fast phenomena in non-scattering regions such as gas combustion (see, for example, [1,2]).

In experimental systems, light is guided by fibre optics to the surface of the subject and detecting fibres are used to measure the transilluminated light. Thus the inverse problem is one of the recovery of coefficients in a domain from data on its boundary, and in many aspects can be considered to be similar to other well-developed fields of research. There are several aspects of the problem however, that make it unusual, and provide a potentially rich set of research topics:

1) The data can be acquired either as time-varying intensities giving the system response to an ultra-short input pulse, or as steady-state complex intensity with measureable amplitude and phase. Because there is a high degree of control available

over the distribution in space and time (or frequency and phase) of the sources, there is the potential to design optimal acquisition systems in a realizable fashion.

2) The forward problem can be interpreted in a variety of ways, either as a particle or wave phenomenon. . In the latter framework, the governing equations can actually be set up in any of a continuum of models that range from a transport model at one extreme to a parabolic (or elliptic) partial differential equation at the other extreme.

3) *A* variety of different coefficients can be considered in the inverse problem, and the degree to which any of these are significant appears to be critical in interpreting the usefulness of the results.

II. BACKGROUND

By optical (diffusion) tomography (OT) we mean the use of visible or near infrared light (NIR) in the wavelength range ~ 700-100 nm, to probe *highly scattering* medium in order to derive images of the optical properties within the medium [3].

In the measurement setup of optical tomography *S* optic fibers are placed on the source positions $\varepsilon_k \subset \partial\Omega$ on the boundary of the body Ω and M optic fibers are placed in the detector positions $\zeta_i \subset \partial\Omega$, see Fig. 1.

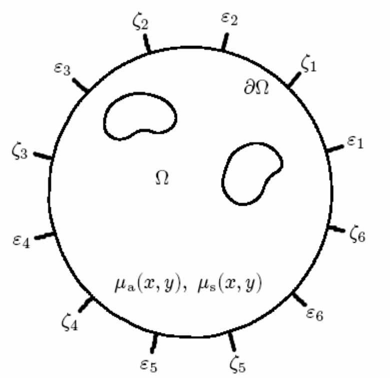

Figure 1: A simple illustration of the measurement setup of OT. The normal lines on the boundary $\partial\Omega$ denote the optic fibres which are attached on the source locations ε_k and detector locations ζ_i

The methods of data acquisition in optical tomography can be divided to three classes which are continuous wave (CW) systems, frequency domain (FD) systems and time-resolved (TR) systems. In all of these systems the basic idea of the data acquisition is the same: Light from a laser source is guided to the body via one of the source fibers at ε_k and the amount of transmitted light is measured on all the detector locations ζ_i, $i = 1,\ldots,M$ using the detector fibers and light sensitive detectors. Then this process is repeated for all the *S* source locations.

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The CW systems use steady state light source and measure the transmitted intensity $\Gamma_i^{(k)}(\omega=0)$ at the measurement sites. In the TR systems the input is an ultra short (duration ~ 10 ps) laser pulse and the measured quantity at the measurement sites $\zeta_{i,}$ is the temporal distribution $\Gamma_i^{(k)}(t)$ of the transmitted photons. A schematic representation of the measurement $\Gamma_i^{(k)}(t)$ between one source and detector location is shown in the bottom row of Fig. 2.

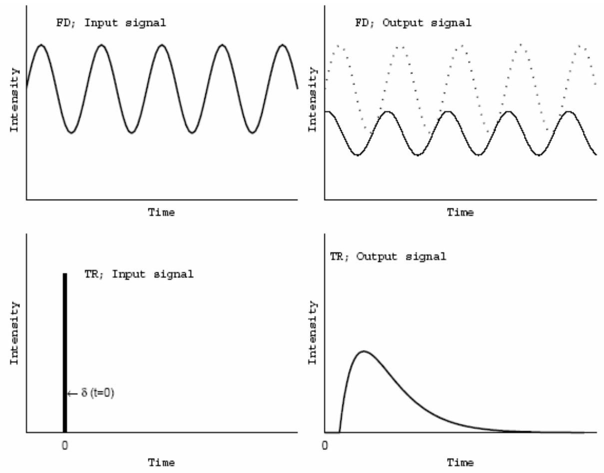

Figure 2: Schematic representation of the OT measurement between one source location ε_k and one detector location ζ_i. Top row illustrates the input (left) and output (right) signals for the FD case. The measured quantities are the modulation amplitude and the phase shift of the transmitted light. For comparision the input is shown with dotted line in the output image. Bottom row illustrates the input pulse and the output (TPSF) for the TR case. In the computations, the ultra short input pulse is approximated with the Dirac delta $\delta(t)$.

In the FD systems, the input is light from a sinusoidally modulated (modulation frequency typically in the range ~ 10MHz - 1GHz) laser source and the measured quantities at the measurement sites are the modulation amplitude $\left|\Gamma_i^{(k)}(\omega)\right|$ and the phase shift $\arg(\Gamma_i^{(k)}(\omega))$ of the transmitted light. An illustration of the frequency domain measurement $\Gamma_i^{(k)}(\omega)$ between one source and detector location is shown in the top row of Fig. 2.

III. RADIATIVE TRANSFER EQUATION

A common basis for the computation of the propagation of light in biological tissues is the radiative transfer equation (RTE).

For monochromatic light, the time-dependent RTE assumes the form

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+s\nabla+\mu_s+\mu_a\right)\phi(r,t,s)=$$
$$=\mu_s\int\phi(r,t,s)\Theta(s,s')ds'+q(r,t,s)$$

where the scalar field $\phi(r,t,s)$ is the energy radiance, the parameters μ_a and μ_s are the absorption and scattering coefficients of the medium, respectively, c is the speed of light in the medium, s is unit vector in the direction of interest and q(r,t,s) is the source term within Ω .

The frequency domain version of the diffusion approximation radiative transfer equation is

$$-\nabla k\nabla\Phi(r,\omega)+\mu_a\Phi(r,\omega)+\frac{i\omega}{c}\Phi(r,\omega)=q_0(r,\omega)$$

Consider diffuse equation:

$$-div(pgrad(u))+(\mu+\frac{i\omega}{v})u=\delta(r) \qquad (1)$$

Equation (1) can be written in the equivalent form [4]

$$\begin{cases}-div(pgrad(u_1))+\mu u_1-\frac{\omega}{v}u_2=\delta(r)\\ -div(pgrad(u_2))+\mu u_2-\frac{\omega}{v}u_1=0\end{cases} \qquad (2)$$

where u_1 and u_2 is real and imaginary parts of complex function u respectively.

The function u_1 and u_2 are expressed in the form

$$u_1=\sum_j q_j^1\phi_j \qquad (3)$$

$$u_2=\sum_j q_j^2\phi_j \qquad (4)$$

By substituting equation (3) and (4) into variational formulation for equation (2), we obtain SLAE for q_j^1 and q_j^2

$$\begin{cases}\sum_j\int_\Omega pgrad(\phi_i)grad(\phi_j)d\Omega\cdot q_j^1+\int_\Omega\mu\phi_i\phi_j d\Omega\cdot q_j^1-\\ \qquad -\frac{\omega}{v}\int_\Omega\phi_i\phi_j d\Omega\cdot q_j^2=\int_\Omega\delta\phi_i d\Omega\\ \sum_j\int_\Omega pgrad(\phi_i)grad(\phi_j)d\Omega\cdot q_j^2+\int_\Omega\mu\phi_i\phi_j d\Omega\cdot q_j^2+\\ \qquad +\frac{\omega}{v}\int_\Omega\phi_i\phi_j d\Omega\cdot q_j^1=0.\end{cases} \qquad (5)$$

In the computations the domain was discretised to N triangular elements Ω_m . The nodal basis functions of FEM mesh are piecewise linear basis function.

Defined a_{ij} and b_{ij} through integrals of system (5)

$$a_{ij}=\int_\Omega grad(\phi_i)grad(\phi_j)d\Omega \qquad (6)$$

$$b_{ij}=\int_\Omega\phi_i\phi_j d\Omega \qquad (7)$$

The local matrix of finite elements Ω_m is obtained formally as

$$A=\begin{pmatrix} p\cdot a_{11}+\mu\cdot b_{11} & -\frac{\omega}{v}\cdot b_{11} & \cdots & \cdots & \cdots & \cdots \\ \frac{\omega}{v}\cdot b_{11} & p\cdot a_{11}+\mu\cdot b_{11} & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix} \quad (8)$$

The coefficient in decomposition basis function can be written as

$$\begin{pmatrix} \alpha_0^1 & \alpha_1^1 & \alpha_2^1 \\ \alpha_0^2 & \alpha_1^2 & \alpha_2^2 \\ \alpha_0^3 & \alpha_1^3 & \alpha_2^3 \end{pmatrix} = D^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ \overline{x}_1 & \overline{x}_2 & \overline{x}_3 \\ \overline{y}_1 & \overline{y}_2 & \overline{y}_3 \end{pmatrix}^{-1} . \quad (9)$$

We can finally write elements of local matrix A

$$a_{12}=a_{34}=a_{56}=\left|\det D\right|\left(-\frac{\omega}{v}\mu/12\right), \quad (10)$$

$$a_{21}=a_{43}=a_{65}=\left|\det D\right|\left(\frac{\omega}{v}\mu/12\right). \quad (11)$$

To solve system of equation by using Biconjugate Gradients method.

IV. INVERSE PROBLEM

The inverse problem in OT is to find estimates for the unknown absorption and scattering coefficients (μ_a,μ_s) within Ω when the distribution of the light sources and the measured data z on $\partial\Omega$ are given Obviously, within the full transport model the task is to find the coefficients of radiative transfer equation, and within the diffusion approximation framework the task is to find coefficients of an elliptic or an parabolic PDE. In this case, we concentrate on the diffusion approximation case.

Most of the current approaches to the optical tomography inverse problem are based on the nonlinear LS approach.

LS objective functional for the OT inverse problem can be stated as

$$\Xi(k,\mu_a)=\left\|L_\omega(z-F(k,\mu_a))\right\|^2 . \quad (12)$$

The most commonly used algorithms for minimizing the objective functional (12) are the gradient methods such as the nonlinear conjugate gradient method and the Newton type methods. In this paper we use the Levenberg-Marquardt method.

At this stage, we specify the discretization for the coefficients (k,μ_a). The coefficients k and μ_a are in most cases expressed in the form

$$k(r)=\sum_{j=1}^{M_k} k_j\chi_j^{(k)}(r)\in H_{M_k}^{(k)} \quad (13)$$

$$\mu_a(r)=\sum_{j=1}^{M_\mu} \mu_{a,j}\chi_j^{(\mu)}(r)\in H_{M_\mu}^{(\mu_a)} \quad (14)$$

The functions $\chi_j^{(k)}$ and $\chi_j^{(\mu)}$ are typically some local-basis functions, such as the characteristic functions of disjoint pixels or the nodal basis functions of the finite element mesh.

Within the expression (13)-(14), we identify the coefficients (k,μ_a) with the vectors

$$k=(k_1,...,k_{M_k})^T \in R^{M_k}$$

$$\mu_a=(\mu_{a,1},...,\mu_{a,M_\mu})^T \in R^{M_\mu}$$

and the parameter vector for the inverse problem becomes

$$\begin{pmatrix} k \\ \mu_a \end{pmatrix}\in R^{M_k}\times R^{M_\mu} .$$

Although it is possible to use different resolution for the coefficients k and μ_a they are usually expressed in similar basis, that is, $M_\mu=M_k=M$ and the basis functions $\chi_j^{(k)}(r)$ and $\chi_j^{(\mu)}(r)$ are the same functions.

Consider the choice of the initial guess for the Levenberg-Marquardt method. If prior information on the internal structure of the domain Ω is not available, the initial guess for the absorption and diffusion coefficients is typically chosen to be constant. These constants may be, for example, the values of skin which are measured from a skin sample by other means.

Once the initial value is chosen, the Levenberg-Marquardt iteration step

$$\begin{pmatrix} k \\ \mu_a \end{pmatrix}^{(k)} \to \begin{pmatrix} k \\ \mu_a \end{pmatrix}^{(k+1)}$$

is computed formally as

$$\begin{pmatrix} k \\ \mu_a \end{pmatrix}^{(k+1)} = \begin{pmatrix} k \\ \mu_a \end{pmatrix}^{(k)} + \\ +s_{(k)}(J_{(k)}^T W J_{(k)}+\lambda I)^{-1} J_{(k)}^T W(g-F(k^{(k)},\mu_a^{(k)})) \quad (15)$$

where $W=L_w^T L_w$, $s_{(k)}$ is the step length parameter and the parenthesized super and subindices denote the iteration index.

To complete the discussion on the nonlinear LS approaches to optical tomography, we derive the Jacobian matrix J in the iteration (15). Consider the case in which the functions (k, μ_a) are approximated in the form (13)-(14) and the basis functions $\chi_m^{(k)}(r)$ and $\chi_m^{(\mu)}(r)$ some local basis functions such as the characteristic functions of disjoint pixels. With these assumptions, the Jacobian assumes the form

$$J = \begin{pmatrix} \dfrac{\partial F_1(k,\mu_a)}{\partial k_1} & \cdots & \dfrac{\partial F_1(k,\mu_a)}{\partial \mu_{a,M_\mu}} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial F_s(k,\mu_a)}{\partial k_1} & \cdots & \dfrac{\partial F_s(k,\mu_a)}{\partial \mu_{a,M_\mu}} \end{pmatrix}$$

where $F_k(k, \mu_a)$ is the forward mapping for the k source.

To derive the elements of the matrix J, we note that using the approximations (13)-(14) and denoting

$$K_{i,j} = \sum_{m=1}^{M_k} k_k K_{i,j}^{(m)} \qquad (16)$$

$$C_{i,j} = \sum_{m=1}^{M_\mu} \mu_{a,k} C_{i,j}^{(m)} \qquad (17)$$

where

$$K_{i,j}^{(m)} = \int_\Omega \chi_m^{(k)} \nabla\varphi_i \nabla\varphi_j dr \qquad (18)$$

$$C_{i,j}^{(m)} = \int_\Omega \chi_m^{(\mu)} \varphi_i \varphi_j dr \qquad (19)$$

Next, differentiating the FEM approximation diffuse equation with respect to the coefficients $k_m, \mu_{a,m}$ and using the expansion (16)-(19), we arrive at

$$(K(k) + C(\mu_a) + R + i\omega Z)\frac{\partial \Phi^h}{\partial k_m} = -K^{(m)}\Phi^h$$

$$(K(k) + C(\mu_a) + R + i\omega Z)\frac{\partial \Phi^h}{\partial \mu_{a,m}} = -C^{(m)}\Phi^h$$

V. NUMERICAL RESULT

In this section, we present some numerical results of simulations performed using the C++ program we developed. The measurement system that the simulation is based on, consists of 1 source fibers and 155 detector fibers in equiangular positions $\varepsilon_1, \zeta_1, ..., \zeta_{155}$ on boundary $\partial\Omega$ of a square body Ω which has side 205 mm. In the construction of the phantom domains the values of the optical coefficients were chosen from the range of interest in medical imaging. For the absorption coefficient μ_a we used values from the range 0.025 (1/mm) – 0.05 (1/mm) and for the diffusion coefficient k we used values from the range 2 mm – 4 mm. The simulated data was computed by using a single (angular) frequency $\omega = 300 \cdot 10^6\, rad/s$.

In the image reconstruction process we used a mesh which consisted of $N_e = 3362$ triangular elements with the total number of $N_n = 1764$ vertex nodes. The results are shown in Fig. 3 –Fig.5.

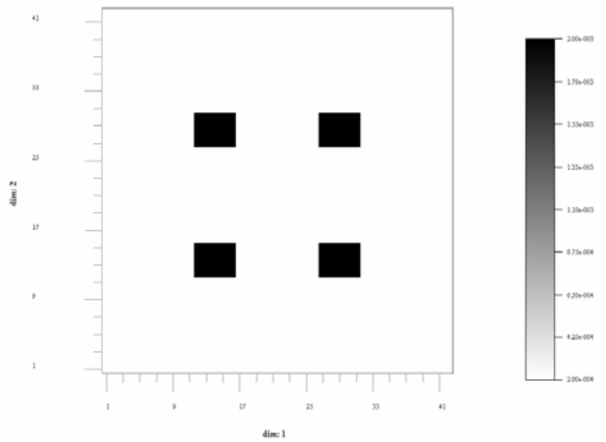

Fig. 3. True images: k

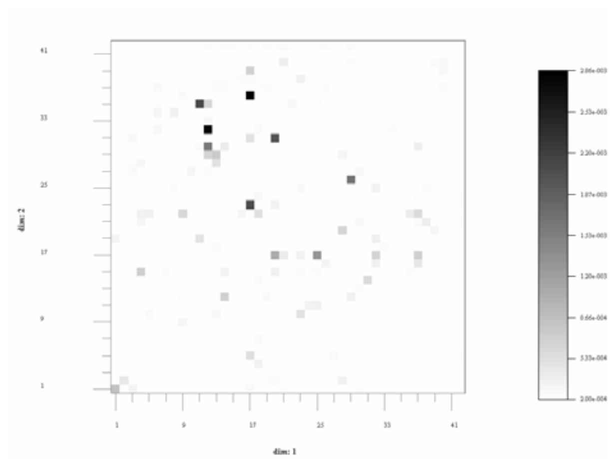

Fig. 4. Reconstructions into a local pixel basis, 1 iteration

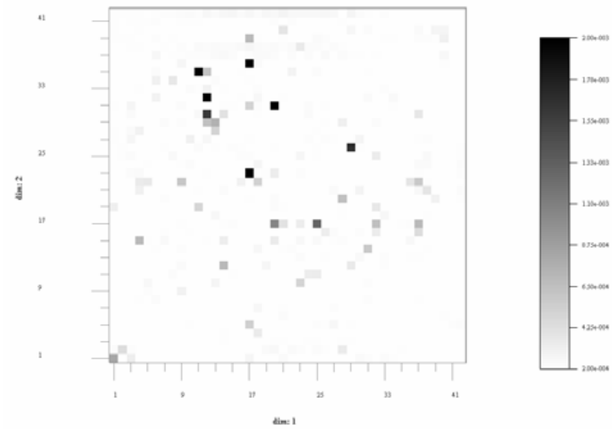

Fig. 5. Reconstructions into a local pixel basis, 2 iteration

VI. CONCLUSION

Optical tomography presents many challenges of interest to theoreticians and experimentalists alike. Optical tomography is a complex and fast-moving area. Our intention here was to present an overview of the methods developed for the inverse problem. In optical tomography the inverse problem is ill-posed and the inability to recover high resolution details is most probably because the latter are in the null-space of the diffusion operator.

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