Numerical Statistical Phisics Metods in Medical Images Analisys Problem

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Abstract - **In our work we consider using Ising and Heisenberg models in problems of increasing quality and analysis medical images. The purpose of our work is research of computer algorithm's efficiency. Computer program we have developed is enable to research medical images operating methods of statistical physics.**

I. INTRODUCTION

Simulation annealing algorithms were created to solve problems of statistical physics. Large physical systems during slow cooling aspire to minimal energy condition. This condition are called basic. Usually basic conditions have high orderliness, for example, crystals or ferromagnetics. It is necessary to note that the cooling speed it is very important. For example, if melted quartz is freezed too fast then amorphous body quartz glass corresponding to a metastable condition turns out, instead of crystals corresponded to stable condition. The methods developed for statistical physics, it is possible to use for improvement of quality and the analysis of images in various areas. In our work we show application Heisenberg model in medical images analysis and Ising model for increasing quality noisy images. The data obtained as a result of computer simulation shows that for images with neoplastic processes corresponded less values of Renie dimensions.

II. SIMULATION ANNEALING ALGORITHM

As known algorithms of imitation annealing rush a global minimum and are not dumped in a local minimum. One of such algorithms is the Metropolis algorithm. They have an extensive scope. Metropolis algorithm have well recommended themselves in models of "low temperatures" and a plenty of statuses. Let's designate for *H* - power function. The current configuration *x* varies on the following algorithm.

Modeling of updating: first we create a trial configuration *y* on the some probability to distribution *G (x, y).*

2. Acceptance or rejection of updating *y*
   a. If $H(y) \le H(x)$, then *y* is accepted as a new configuration
   b. если $H(y) > H(x)$, then *y* is accepted as a new configuration with probability $exp(H(x)-H(y))$
   c. If updating *y* deviates, there is an old configuration *x*

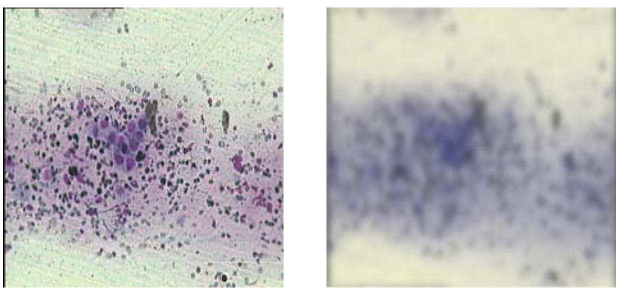
Matrix *G (x, y)* - matrix of updating. Using this algorithm it is possible to model Gibbs field with power function *H*. Status *y* with the greater energy, than energy *x*, is not rejected at once, [1]and it can be accepted with probability *exp (H (x)-H (y))*. In simulation annealing algorithms this method allows to leave points of a local minimum. At the analysis of images natural procedure of updating will be to choose an any pixel of the image, and then in the casual mode to change its value. Formally it can be written down as follows:

$$G(x,y)=\begin{cases}\dfrac{1}{q(N-1)}, & if\ \ x\neq y,\ \ exactly\ in\ one\ site\\ 0,\ else\end{cases} \tag{1}$$

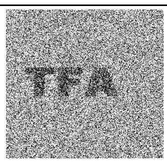
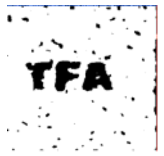
Where *q* - number of pixels, and *N* - set of possible statuses of a pixel.

Algorithms with such matrix of updating are named algorithms with (equiprobable) one-dot updatings. For the majority of images *N* - very big. The choice of a new status will be, that in the casual mode to choose *y* and then if $H(y) \le H(x)$ to check up probability of transition in a new status. At local dependence of energy on a configuration this procedure is not too toilful, as against modeling local characteristics in Gibbs method. That speaks about efficiency of a method of the Metropolis.

More often noise in the image results in small-scale distortions, certainly, if it completely does not destroy the information. Since real images will usually consist of homogeneous sites methods of restoration of images that or a different way smooth the image. Smoothing possesses undesirable property to wash away border and to reduce picture contrast. In the following figure the example of the usual and smoothed image is resulted.



[1] Investigation partially supported by RFFI grants 05-01-00171, 05-01-00559 and grants SS-1172.2003.1, UR-04.01.200.

Noise	Initial picture	Noise image	Reconstructed picture
31%			
46%			
62%			

Let's designate a status, in which pixel black for 1, and when it white for 0. It will allow to use 2 dimensional Ising model. To black pixel corresponds spin of Ising model directed upward, white - downward. Using Metropolis algorithm for such model, it is possible to restore the image. Power function in this case

$$H(x) = \beta \sum_{<i,j>} x_i x_j \tag{2}$$

Where $<i, j>$ - means, that sites i and j are neighbour,

$$\beta = -\frac{J}{kT} \tag{3}$$

As a result of algorithm work, we have received the image with some distortions on borders, but such image is already much easier for distinguishing.

III. THE ANALISIS OF IMAGES

In most cases for medical images color and character of borders is were important. It is known, that borders of malignant tumours have fractal character. However, when figure color the number of possible conditions is very big. For the analysis of the color image we used the following model. Each color pixel can put in conformity its decomposition on three basic colours in palette RGB. The range of change on each color of a palette lays on a piece [0,255]. Each piece in turn can compare a corner of turn on a circle. Such approach allows us to proceed to Heisenberg model. In such model everyone of backs can be not in two possible conditions as in Ising model, and in trusts sixty. Such approach allows us to investigate color images.

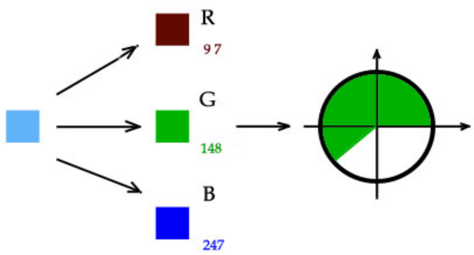


Fig. 2. Transition from the image to Hesinberg model

Let's describe interaction spins. The new configuration y turns out by means of turn on 180^0 random spin, and this turn occurs on each of colors. For each new configuration we calculate a corner between a new condition spin and a total corner of its neighbours. If the difference on the module is less 120^0 that this configuration is accepted. If the corner is more, transition in a new condition is carried out with probability

$$\exp\left(\left(\frac{f-g}{360}-1\right)\Big/T\right) \tag{4}$$

where f a new corner, g a total corner of neighbours.

We break area which occupies our image, into cells in the size e. Let the number of spins in a cell is equal $n_i(e)$, and the number of occupied cells $N(e)$.

Then the probability, that that at random spin contains in i to a cell will be equal

$$p_i(e) = \lim_{N\to\infty} \frac{n_i(e)}{N} \tag{5}$$

Generalized statistical sum $Z(q,e)$, is determined by the formula

$$Z(q,e) = \sum_{i=1}^{N(\varepsilon)} p_i^q(e) \tag{6}$$

The spectrum generalized fractal dimensions D_q, describing the given distribution of points, is defined by a ratio

$$D_q = \frac{1}{q-1} \lim_{e\to 0} \frac{\ln Z(q,e)}{\ln e} \tag{7}$$

At $q=0$ it is received Haustdorf dimension. At $q=1$ it is received

$$D_1 = \lim_{e\to 0} \frac{\sum_{i=1}^{N(e)} p_i \ln p_i}{\ln e} \tag{8}$$

To within a sign the above formula represents entropy. Dimension D1 is the dimension of information. Such algorithm and spectrum of dimensions, allows to investigate Hesinberg model. We studied medical photos with tumours and normal cells. For each image dynamics of change of dimensions D1, D2, D30 has been constructed depending on temperature.

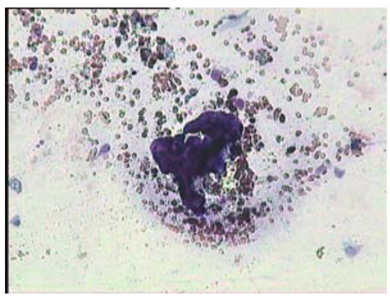


Fig. 3. Example of the image with a tumour

In the following figure the schedule of dimension D_1 is shown.

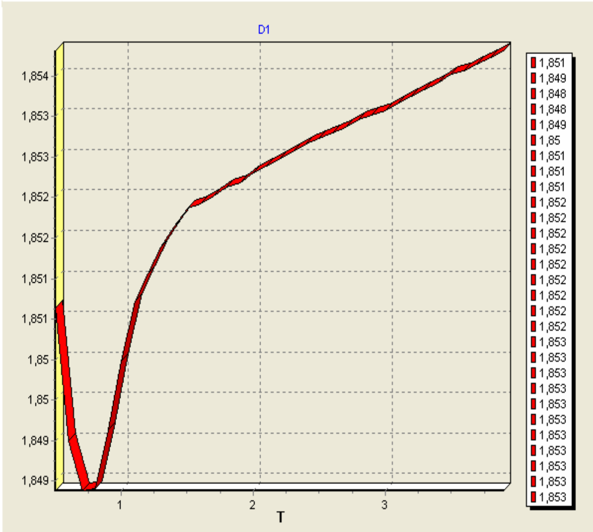


Fig. 4. Schedule of dependence of dimension D_1 from temperature

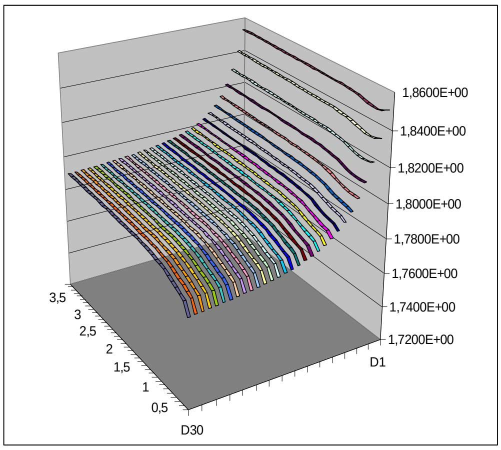


Fig. 5. Dependence of dimensions on temperature

IV. CONCLUSION

Besides we had been investigated first thirty dimensions of the image on a temperature interval [0.5,3.5].

In fig. (5) their communication with temperature is represented.

For diagnostics neoplastic processes in tissues usually use neural networks. However their lack is long time which the neural network spends for recognition and the analysis of the image. At the initial stage of preparation of the information to a neural network apply various filters. They sort some part of images as obviously pathological or on the contrary healthy. Such approach allows to speed up time of processing. The case when the program cannot give the precise conclusion about presence or absence of a pathology should be processed in addition a neural network. The program developed by us is just such filter. With the help of fractal dimensions determined at various temperatures of Heisenberg model changes in model are traced. Dynamics of these changes is traced by the program, whether and on its basis is concluded that the given image to neoplastics belongs to processes, or the pathological image takes place not.

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