Qualitative Research of Mathematical Model for Future Markets and Prediction Opportunity of Trends Changing

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Abstract - **This paper gives quick overview of future markets mathematical model. Qualitative research of differential equations system is carried out. Correlation of trends and special points' trajectories is revealed.**

I. INTRODUCTION

In our days, methods of the determined chaos are intensively developed at modeling economic processes [3]. One of the most perspective appliances of these methods is researches in the field of forecasting dynamics of market characteristics.

Dynamic model for future markets was offered in [3], one of its advantages is reception forecasting realizations of economic characteristics in view of their mutual influence. This model allows to receive quantitative ratings - affinity of predicted parameters to the real data; horizon of the forecast; a way of forecast specification.

For development of this model and disclosing of its potential opportunities and advantages in the description and forecasting of market characteristics is necessary to carry out mathematical researches of nonlinear order differential equations (ODEs) system. It will allow determining structure of forecast mistake and the most effective circuits of adaptation of model.

This work based on ODE system solutions analysis and allocation trend and chaotic components. The technique for definition of the trend change moments, improving efficiency of the forecast is offered.

II. QUALITATIVE RESEARCH OF ODE SYSTEM

In future markets model exchange information is considered as the determined chaos, i.e. chaotic change of parameters is irregular (chaotic), generated by nonlinear systems, for its dynamic laws unambiguously determine evolution on the chosen time interval Δt ($\Delta t / T << 1$, Δt - corresponding trading session, T - length of researched time series) at known background [2]. The basic future markets model equations are submitted in the matrix form as in [3]:

$$\dot{\bar{X}} = \boldsymbol{A}\bar{X} + \bar{F}\ , \qquad (1)$$

$$\boldsymbol{A} = \begin{bmatrix} a_1(t) & 0 & 0 \\ 0 & b_2(t) & 0 \\ 0 & 0 & c_3(t) \end{bmatrix},\ \bar{X} = \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{bmatrix},$$

$$\bar{F} = \begin{bmatrix} a_2(t)X_1(t)X_2(t) + a_3(t)X_1(t)X_3(t) \\ b_1(t)X_1(t)X_2(t) + b_3(t)X_2(t)X_3(t) \\ c_1(t)X_1(t)X_3(t) + c_2(t)X_2(t)X_3(t) \end{bmatrix},$$

where system parameters are price (X_1), tender volume (X_2) and «open interest» (X_3). The parameters interrelation is represented in cross products [1]. Factors a_i, b_i, c_i ($i = \overline{1,3}$) determine influence degree of model components.

Let's apply methods of ODE qualitative theory for more detailed research of future market model. First, shall find system balance points and then consider system movement near each balance position. It is known, that for first-order ODE system $\dot{\bar{X}} = f(\bar{X})$, balance points are determined by equality:

$$\dot{\bar{X}} = 0 \quad \text{or} \quad f(\bar{X}_b) = 0\ , \qquad (2)$$

where $\bar{X}$ a vector of system status.

Second, we must spread out function $f(\bar{X})$ in Taylor series near to each balance point $\bar{X}_b$ for finding-out of solution behaviour character near to these points and consider linearized problems. Movement character near to each balance points is found out with eigen values λ_i of the characteristic polynomial. Stability of linearized system solution is determined with a sign of the real part $\mathrm{Re}(\lambda_i)$: when the real part at least one of eigen values λ_i is positive, movement near to this balance point is unstable [4].

The equations (2) for future markets model (1) describing balance position are:

$$\begin{cases} 0 = a_1(t)X_1(t) + a_2(t)X_1(t)X_2(t) + a_3(t)X_1(t)X_3(t), \\ 0 = b_1(t)X_2(t)X_1(t) + b_2(t)X_2(t) + b_3(t)X_2(t)X_3(t), \\ 0 = c_1(t)X_3(t)X_1(t) + c_2(t)X_3(t)X_2(t) + c_3(t)X_3(t). \end{cases}$$

Solution of this system is five balance points on each moment of time:

$$O_1 = [0 \quad 0 \quad 0], \text{ - trivial solution}$$

$$O_2 = \begin{bmatrix} 0, \\ -c_3/c_2, \\ -b_2/b_3, \end{bmatrix} \quad O_3 = \begin{bmatrix} -c_3/c_1, \\ 0, \\ -a_1/a_3, \end{bmatrix} \quad O_4 = \begin{bmatrix} -b_2/b_1, \\ -a_1/a_2, \\ 0, \end{bmatrix}$$

$$O_5 = \begin{bmatrix} \dfrac{-a_2b_3c_3 - a_3b_2c_2 + a_1b_3c_2}{a_2b_3c_1 + a_3b_1c_2}, \\ \\ \dfrac{-a_3b_1c_3 + a_3b_2c_1 - a_1b_3c_1}{a_2b_3c_1 + a_3b_1c_2}, \\ \\ \dfrac{-a_1b_1c_1 + a_2b_1c_3 - a_2b_2c_2}{a_2b_3c_1 + a_3b_1c_2}, \end{bmatrix}$$

where $a_i, b_i, c_i \quad (i = \overline{1,3})$ - model factors on the considered time interval.

Let's make out the characteristic equation on linearized systems factors of the equations (1) for determination of balance points $\overline{X}_b = [X_{1b} \quad X_{2b} \quad X_{3b}]$ character and for finding out eigen values λ_i:

$$\det(\mathbf{B} - \lambda \mathbf{E}) = 0,$$

where **B** - matrix of corresponding linearized systems factors in balance point, **E** - unitary matrix. Characteristic polynomial for submitted model is:

$$\begin{vmatrix} a_1 + a_2X_{2b} + a_3X_{3b} - \lambda & a_2X_{1b} & a_3X_{1b} \\ b_1X_{2b} & b_1X_{1b} + b_2 + b_3X_{3b} - \lambda & b_3X_{2b} \\ c_1X_{3b} & c_2X_{3b} & c_1X_{1b} + c_2X_{2b} + c_3 - \lambda \end{vmatrix} = 0 \qquad (3)$$

Analysis of equation (3) has shown that in domain of real parameters existence all five balance points of model are unstable. The real part $\mathrm{Re}(\lambda_i)$ at least one of eigen values λ_i is positive. It confirms that the financial markets have unstable character, i.e. they subjected to external accidental influences, hence long-term forecasts are less reliable and short-term forecasts have significant advantages.

Let's compare trajectories of special points (balance points) to behaviour of economic characteristics. Thus, not informative solutions to which concern the trivial solution (O_1) for all system parameters, and that solution in which, coordinate corresponding to parameter accepts zero value we shall reject. (For the price is O_2, for tender volume- O_3, for «open interest» - O_4.)

Real trajectories of financial markets parameters are resulted on fig. 1. All data are normalized.

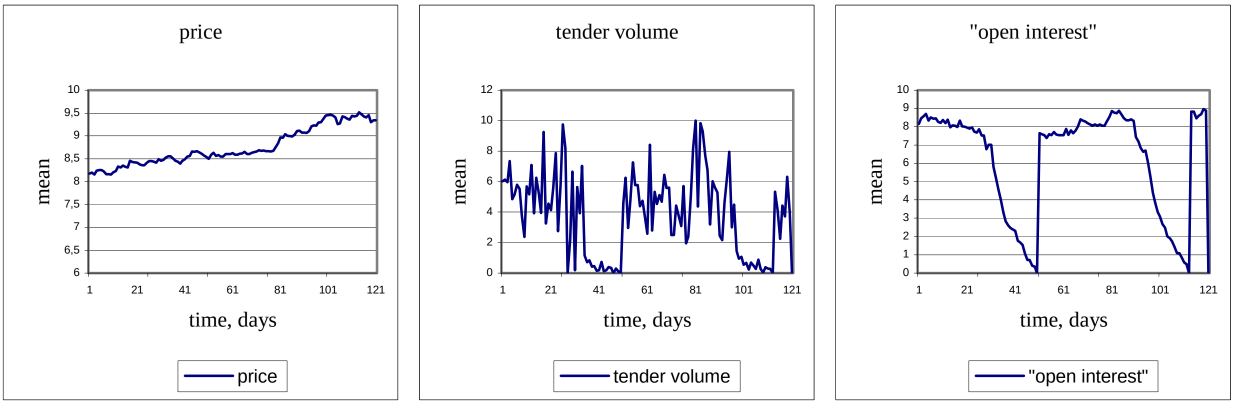

Fig. 1. Real trajectories of financial market parameters.

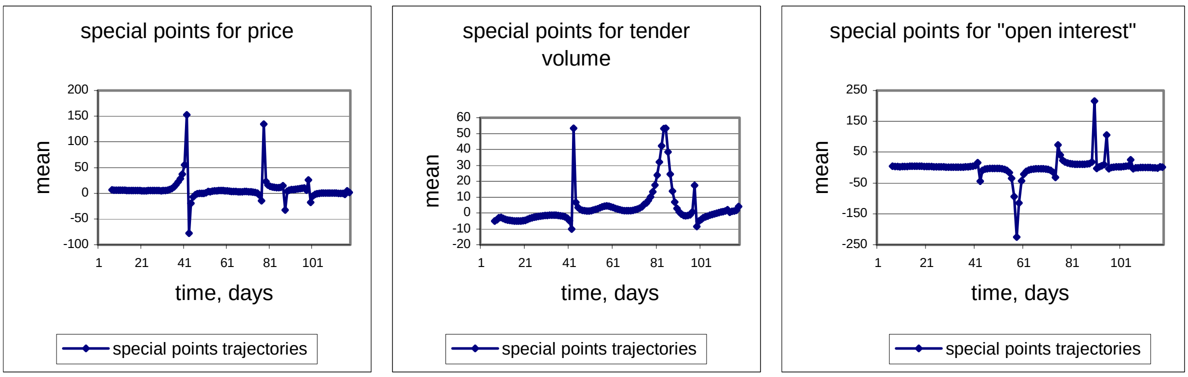

Fig. 2. Coordinates of special points for trend component.

Trajectories of financial markets parameters are broken enough. It prevents to establish correlation between trajectories of special points and real trajectories of the exchange information. Therefore, we must determine correlation of special points' trajectories with the smoothed economic characteristics.

III. ALLOCATION TREND AND CHAOTIC COMPONENTS

It is known from physics, that if system is not influenced with external forces it changes with the frequency determined by system characteristics. Similar situation develops in the financial markets. If formation of parameters is not influenced with any external factors, then parameters start to change with some frequency determined by internal market laws. In any case not strong external enough influences result only in some fluctuation of parameters $\bar{X}$ about table component. In any case, weak enough external influences cause only some parameters fluctuation $\bar{X}$ about a stable component.

Thus, real information ($\bar{X}$) can be presented as sum of two components: trend ($\bar{T}$) and chaotic ($\bar{H}$):

$$X_k = T_k + H_k\,,\ k = \overline{1,3}\,,$$

where k - number of phase coordinate. Trend component is some smoothed curve without sharp chaotic (accidental) pips. For its determination chose moving average and polynomial approximation methods.

Polynomial of mth degree (m $\geq$ 4) used for approximation. Chaotic character of H_k confirmed experimentally, i.e. received data:

- look "accidentally";
- autocorrelation function falls down rapidly;
- power spectrum is represented by continuous wide strip on low frequencies, that responds chaotic character criterion [2, 5], hence, received polynomial is trend. For tender volume and «open interest » if necessary, it is possible to allocate a periodic component.

IV. CORRELATION BETWEEN SPECIAL POINTS AND TRENDS

Let's apply qualitative theory of ODEs to smoothed series (trends). We must find special points and construct trajectories formed by special points for trend component. Results of calculations are depicted on fig. 3. There is an interrelation between trajectories of special points and real values trend components. The most interesting result is prediction opportunity of trend direction changes by behaviour of special points.

Indeed, there are moments in which trajectories of special points change values sharply. Points on real trend curve (fig. 4) in which, change of trend direction happens, corresponds to these moments. This is apparent from submitted results.

For example, (fig. 3.), they are moments 41-42, 84-85, 98-99, 113-114 for tender volume. Let's mark these moments at corresponding smoothed characteristic (fig. 4). After a while, interval trend component will

change movement direction or speed of change, i.e. change of trend direction will happened.

Similar dependence can be noticed for price - points 42-43, 77-78, 87-88, 98-99, 115-116 and «open interest» - points 43-44, 58-59, 75-76, 90, 95, 105-106. Thus, special point pips predict moment of trend change some steps before real change.

For example, this interval S depends on parameter and is equal from 2 till 5 days for submitted values. Choosing different methods for smoothing of economic characteristics, we receive different, but close values. Increasing degree of smoothing polynomial, we receive information with greater detailed elaboration.

V. CONCLUSIONS

It is possible to formulate the following conclusions:

1. Qualitative analysis has shown existence of five balance positions corresponding linearized system, and all of them are unstable. That confirms fact of market

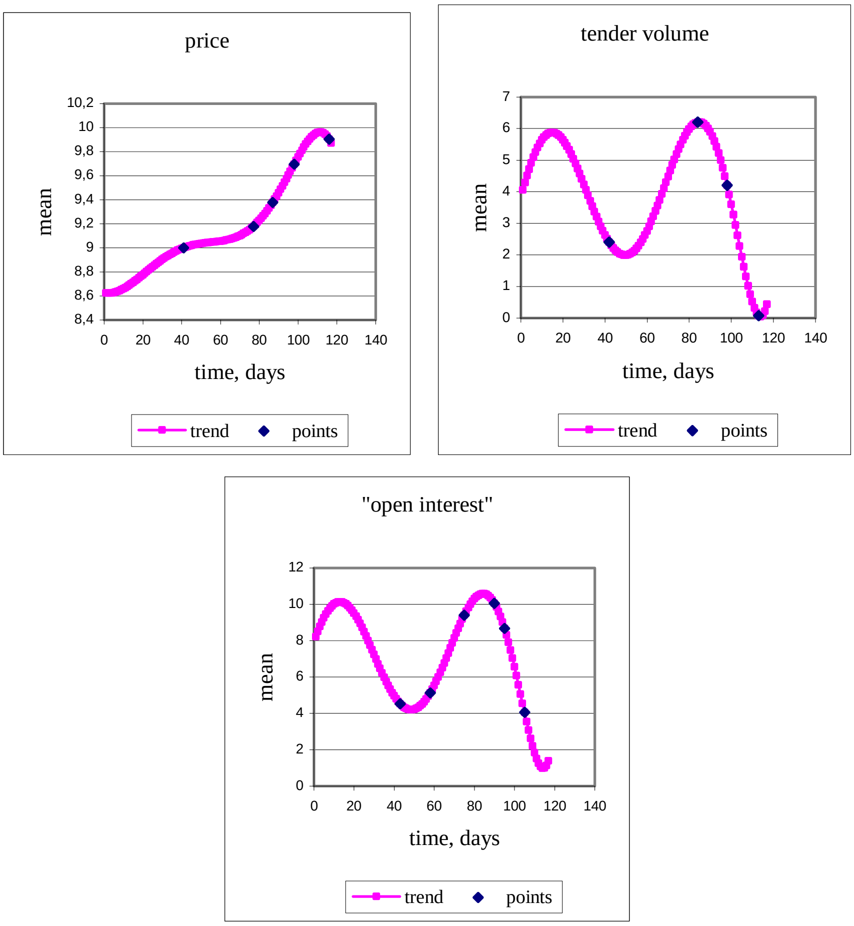

Fig. 3. Trend components with the allocated points of trend change

instability as a whole. Hence, long-term forecasts are less reliable and short-term forecasts have significant advantages.

2. Correlation between trend component values and trajectory of special points for corresponding linearized ODE systems is noticed. This dependence allows to predict trends change and to formalize procedure of decision-making.

Thus, qualitative research of ODE has shown, that there are some latent mechanisms, which determine system behaviour in future. New prediction opportunities of characteristics change tendencies have appeared.

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