Radiation Pattern Synthesis for Arrays Based on Sierpinski Gasket

A.N. Bondarenko, Y.V. Mikhailova.

NSTU
630092, Novosibirsk, K.Marks, 20, Russia
e-mail: bondarenkoan1953@mail.ru, myv24@ngs.ru

***Abstract* - There are considered some problems of fractal antenna array synthesis. Fractal antenna array was considered as systems of the discrete radiator situated along some curve on a plane and the new mathematical device of atomic functions was applied.**

I. INTRODUCTION

There are considered some problems of fractal antenna array synthesis. Fractal antenna array was considered as systems of the discrete radiator situated along some curve on a plane and the new mathematical device of atomic functions was applied.

Recent efforts by several researchers around the world to combine fractal geometry with electromagnetic theory have led to a plethora of new and innovative antenna synthesis. The elementary mathematical abstraction of self-similar set is the Cantor set, its planar analogue is Sierpinski gasket. Some antenna arrays were constructed on the basis of such sets and their properties are investigated.

The first application of atomic functions to the antenna theory was reported by V.F. Kravchenko [1]. His used the behavior of atomic function in analysis of some type of traditional and fractal antennas.

II. STATEMENT OF ARRAY ANTENNA SYNTHESIS PROBLEM

In general case the array antenna represents system of the discrete radiator elements located in space under the certain law. It realize signal processing assumed by elements, or addition in space of the energy received by these elements. In the most research work considered problems of antenna synthesis the interference effect between antennas element was neglected. As practice shows, such approach appears quite acceptable for the decision of many practical problems. In this case the signal on output of the array represents a linear combination of the signals E_n accepted by its elements:

$$E_\Sigma = \sum_{n=1}^{N} E_n . \qquad (1)$$

When plane wave fall on antenna aperture,

$$E_n = A_n D_n(\theta,\varphi)\exp\left[ik\sin\theta\left(x_n\cos\varphi + y_n\sin\varphi\right)\right] \qquad (2)$$

where $A_n(x_n, y_n)$ is a current in the n radiator; $D_n(\theta,\varphi)$ is the radiation pattern of n radiator x_n, y_n is coordinates of position of radiator on a plane; n is a serial number of a radiator.

Usually the radiation pattern represent through the generalized coordinates connected to corners of spherical coordinates:

$$u = k\sin\theta\cos\varphi . \; v = k\sin\theta\sin\varphi . \qquad (3)^1$$

In the generalized coordinates the radiation pattern of a planar array becomes:

$$D(u,v) = \sum_{n=1}^{N} A_n D_n(u,v)\exp\left[i\left(ux_n + vy_n\right)\right]. \qquad (4)$$

Apparently, the array radiation pattern represents complex-valued function of two real variables u and v. If the array is consist of identical radiator,

$$D(u,v) = D_0(u,v)\sum_{n=1}^{N} A_n(x_n, y_n)\exp\left[i\left(ux_n + vy_n\right)\right]. \qquad (5)$$

The problem of synthesis space-tapered array consists in definition of coordinates of an optimum arrangement of array elements and is important and till now up to the end the not decided(solved) problem. Producing of the narrow radiation pattern is require wide antenna aperture and consequently, and the big number of elements in array. And so, if arrays have equally spaced elements than realization of the fractal array advantages is strongly complicated. Reduction of number of elements probably if to have them widely spaced. But in this case the interference lobes can appear at scanning and they are not differing from the main beam. The problem of the radiation pattern choice of array elements is closely connected to a problem of synthesis space-tapered array because application of various size radiators and with various radiation pattern is possible in this case. In the space-tapered array identical elements are usually used. As separate array elements have a wide orientation than the array pattern is mainly defined by the sum $\sum_{n=1}^{N} A_n(x_n, y_n)\exp\left[i\left(ux_n + vy_n\right)\right]$ which named the array factor. It represents the radiation pattern of the isotropic adiator array, where $D_0(u,v)=1$. The element radiation pattern is cast out as not influencing on the array pattern in the solving of a problem of synthesis.

III. DETERMINISTIC FRACTAL ARRAYS

A rich class of fractal arrays exists that can be formed recursively through the repetitive application of a generating subarray [4-5]. In many cases, the generating subarray has elements that are turned on and off in a certain pattern. A set formula for copying, scaling, and translation of the generating subarray is then followed in order to produce the fractal array. Hence, fractal arrays that are created in this manner will be composed of a sequence of self-similar subarrays. In oother words, they may be conveniently throught of as arrays of arrays.

The array factor for a fractal array of this type may be expressed in the general form

$$F_P = \prod_{p=1}^{P} g(\delta^{p-1}\psi) \tag{6}$$

where function $g(\psi)$ represents the array factor associated with the generating subarray. The parameter δ is a scale or expansion factor that governs how large the array grows with each recursive application of the generating subarray.

Let's consider the planar arrays based on various fractal and self-similar functions. These arrays concern to a class equidistant arrays with non-uniform amplitude distribution. From the practical point of view it means, that power brought from the generator may be a various for different radiators. In this case separate array elements are in various behavior modes and can fail owing to overheating. Therefore it is necessary to reduce a side lobe level. As we known from the theory [3], arrays with non-uniform amplitude distribution have lower use factor of the antenna area in comparison with the arrays having uniform distribution. The formula for radiation pattern calculation for array with $(2M)^2$ elements looks like:

$$F\left[\psi_x,\psi_y\right]=4\sum_{n=1}^{M}\sum_{m=1}^{M} I_{mn}\cos\left[(m-1/2)\psi_x\right]\times$$
$$\times\cos\left[(n-1/2)\psi_y\right] \tag{7}$$

where the I matrix assigned the current distribution in the appropriate points x_n and y_n.

IV. THE ATOMIC FUNCTIONS APPLICATION THEORY IN PLANAR RADIATOR SYNTHESIS

Atomic functions are compactly supported solutions to some function-differential equations [3]:

$$\sum_{k=0}^{N} A_k \frac{d^k}{dx^k} f\left(\frac{x}{a}\right)=\sum_{k=-M}^{M} c_k f(x-k), \quad |a|>1. \tag{8}$$

Accordingly, Fourier transform of atomic functions is given by the infinite product:

$$\Phi(z)=\prod_{n=1}^{\infty}\frac{T(za^{-n})}{P(za^{-n})}, \tag{9}$$

where $T(z)$, $P(z)$ are the trigonometric and algebraic polynomials. Atomic function Fourier transform possess a low level of side lobe which decay faster any degree $|z|^{-k}$, $|z|\to\infty$ (atomic function properties as weight functions are investigated in [3]). For example, the function

$$Up(z)=\prod_{k=1}^{\infty}\frac{\sin\left(2^{-k}z\right)}{2^{-k}z}, \tag{10}$$

is Fourier transform of elementary atomic function $up(x)$, has a side lobe level in 10 times smaller, than function $\sin z/z$ which widely used in antennas synthesis problems. Approximating properties of atomic function Fourier transform are investigated in [3] where it was shown, that the radiation pattern of the linear array with length 2π can be interpolated by series

$$R(\mu)=\sum_k R_0(2k)Up(\pi(\mu-2k)) \tag{11}$$

V. SYNTHESIS OF VOLUNTARY CURVILINEAR RADIATOR

Let's consider the curvilinear radiator situated in a plane $z=0$ on the line $y=L(x)$. The synthesis problem of it is reduced to the solution of the following integrated equations:

$$N_x=\int_{-l/2}^{l/2} F_x(x)\sqrt{1+L'(x)}\exp[ik\sin\theta(x\cos\varphi+y\sin\varphi)]dx$$
$$N_y=\int_{-l/2}^{l/2} F_y(x)\sqrt{1+L'(x)}\exp[ik\sin\theta(x\cos\varphi+y\sin\varphi)]dx \tag{12}$$

The length of a radiator projection onto axis x is designated through l.

These equations are initial for the decision of a synthesis problem of the curvilinear radiator located in a plane $z=0$. It is required to definition of two making vectors of an electric field, which is tangential to the antenna aperture plane (one F_x is polarized in regard to axis x, the other is parallel an axis y), and the radiator forms $L(x)$ on the given radiation pattern $D(\theta,\varphi)$ or on the given functions N_x N_y.

The equations (12) are dependent, and functions N_x, N_y can not be given arbitrarily. They should be subordinated to the certain additional conditions. We shall believe, that $L(x)$ is one-valued function and belongs or to functions with integrable square, or to absolutely integrable functions.

Synthesis of the radiator located on a plane along some curve

The array factor of N isotropic radiators located on a plane $z=0$ along some curve $y=L(x)$, has the following kind:

$$D(\theta,\varphi)=\sum_{n=1}^{N} A_n \exp\left[ik\sin\theta\left(x_n\cos\varphi+y_n\sin\varphi\right)\right] \quad (13)$$

Here A_n is the required field excitation of the radiator n: y_n and x_n is required coordinates of points of an array radiators arrangement on a plane $z=0$. In this connection $y_n=L(x_n)$ and N is a number of array radiators.

VI. CALCULATION OF THE FRACTAL ARRAY RADIATION PATTERN

At the initial stage of works the synthesis problem was computing for the fractal planar arrays based on the Sierpinski gasket, which submitted in works Kravchenko V.F. [3-5]. After similar results reception were considered various any fractal antenna array of similar construction and dimension. Results are submitted in figures.

Modified Sierpinski gasket

Let's considered the Sierpinski gasket as the discrete set of radiators, which is situated on parallel line (Fig. 1). Then we can apply the formule (13).

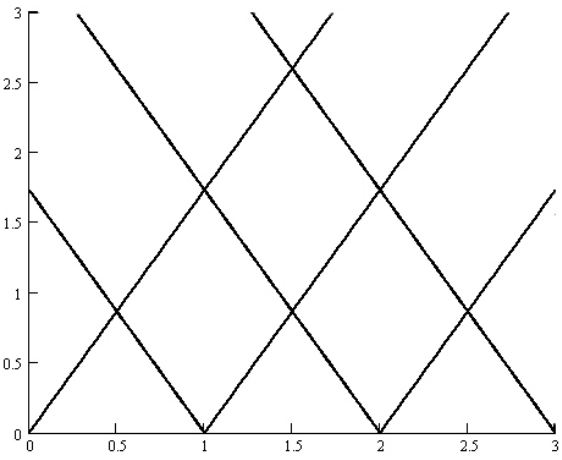

Fig. 1 Some representation of Sierpinski gasket

For simplicity sake at first we modify the traditional Sierpinski gasket in right triangle. The array received thus is considered equivalent the array with the point radiators. Then the given task is solved the methods similar to a problem about Sierpinski carpet [3]. The results submitted in Figure 2 were received. Discrete radiators are located in the centre of light squares.

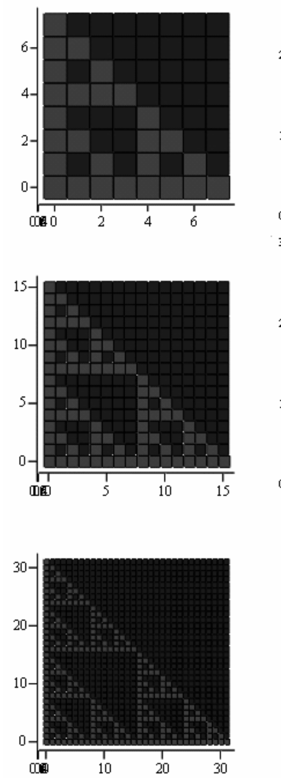

Fig. 2. Planar arrays and its array factors for first three stages of construction.

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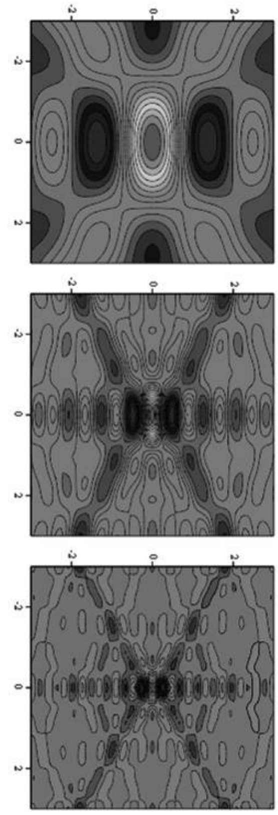

Fig. 3. Array factor of first three stages of Sierpinski gasket construction

Traditional Sierpinski gasket

In case of standard Sierpinski gasket, it is more convenient to present it as the discrete sources located along curvilinear lines on one plane.

Then we shall present array as a set of the parallel lines described by the equations:

$$y_n(x) = \sqrt{3}\left(x - nh\right)$$

Where h - distance between lines on an axis x .

VII. CONCLUSIONS

We considered new methods of the analysis and synthesis fractal antenna array and earlier known problems for the radiators located on a plane along some curvilinear lines. Computer experiments have confirmed the general laws of electromagnetic field distribution and antenna array radiation pattern. Differen variations generating subarrays gave the interesting results demanding the further research.

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